Negative linear classical magnetoresistance in a corrugated two-dimensional electron gas

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Large linear negative magnetoresistance (LNMR) was measured in a GaAs/Al_xGa_{1-x}As two-dimensional electron system with nonplanar topography, caused by random distribution of corrugations, generated by a combination of prepatterning and regrowth processes. Even though the LNMR reaches up to 20% of the zero field resistivity $[\rho_{xx}(B=0)]$, in a very small region around B=0, the resistivity shows a nonanomalous behavior. From comparison with a recent theory development for the conductivity of a classical two-dimensional Lorentz gas, and numerical calculations of the electron dynamics in systems with random electrostatic potentials, performed by us, we argue that the observed MR is mainly due to non-Markovian memory effects originated by specific return processes in backscattering of electrons by corrugations and defects.

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I. INTRODUCTION

Recently the classical low-field magnetoresistance (MR) in metals and semiconductors has been revised since it was recognized that the conventional Boltzmann-Drude approach fails to describe the electron dynamics in disordered systems.^{1–4} Drude result yields zero change of magnetoresistance, in the presence of short range electrostatic potential. This constant resistance, in the presence of uniform magnetic field, results from the exact compensation of the mean free path decrease, in the presence of the Lorentz force, by the Hall effect. In terms of the relaxation time approximation magnetoresistance can be rewritten in the form:

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} = \omega_c^2 (\langle \tau_{tr}^2 \rangle - \langle \tau_{tr} \rangle^2), \qquad (1)$$

where $\langle \tau_{tr} \rangle$ is the average transport time, $\omega_c = eB/mc$ is the cyclotron frequency, *m* is the electron effective mass, and $\rho_{xx}(0)$ is the resistivity at zero magnetic field. We may see that when $\langle \tau_{tr}^2 \rangle = \langle \tau_{tr} \rangle^2$, which is valid in the framework of the time relaxation approximation, the longitudinal magnetore-sistance is zero. For several specific cases, such as scattering in the long range random magnetic potential (RMF), the treatment beyond the relaxation time approximation is necessary.

A new semiclassical approach of the electron dynamics in random magnetic field, in the presence of the uniform perpendicular magnetic field B_{\perp} , has been developed in the models cited in Refs. 5 and 6, where it was obtained negative magnetoresistance of the type

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} = -C\omega_c^2 \tau_{tr}^2,\tag{2}$$

where numerical factor C depends on the realistic RMF realization. Such negative magnetoresitance has been studied in experiments with nonplanar two-dimensional electron gas (2DEG).^{7,8} Due to the presence of interfacial roughness, electrons in such structures see an external uniform in-plane magnetic field B_{\parallel} as a random perpendicular magnetic field $B_{\rm rand}^{\perp,9}$ The presence of an additional small uniform perpendicular field leads to the negative MR in accordance with theoretical predictions.^{5,6}

Another theoretical approach beyond the Boltzmann approximation is associated with so-called memory effects in electronic transport. In the classical Boltzmann-Drude model, all scatterers should be redistributed in a completely stochastic way, after each collision, according to the Stosszahlansatz. This means that no correlation exists between successive scattering events because an electron loses its past memory after a few immediately preceding collisions with other scatterers.^{1,4}

However, in realistic systems scattering is not completely stochastic because there is always a finite probability that electrons may recollide with the same impurity, or that they may not collide with any scatterer. These processes introduce non-Markovian effects into the kinetic Boltzmann-Drude model and produce significative corrections to the conductivity that should be taken into account. This has been demonstrated, more clearly, in a two-dimensional (2D) model that considers noninteracting electrons scattered by hard disks (2D Lorentz model), in such a system magnetoresistance is given by the formula^{1,10,11}

$$\frac{\rho_{xx}(B)}{\rho_{xx}(0)} = 1 - \exp\left(\frac{-2\pi}{\omega_c \tau_{tr}}\right).$$
(3)

Equation (3), that includes both the contributions of "circling" and "wandering" electrons, is valid in the limit $Nd^2 \rightarrow 0$, where N is the 2D concentration of the scattering centers and d is the effective diameter of the disk. For finite values of Nd^2 the resistivity behaves as¹¹

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} = 1 - \frac{0.15}{NR_c^2},\tag{4}$$

where $R_c = v_F / \omega_c$ is the cyclotron radius and v_F is the Fermi velocity. This equation predicts a negative parabolic magnetoresistance at low magnetic field ($\omega_c \tau_{tr} \ll 1$). The conductivity of the 2D Lorentz model has been corrected, further, by a more realistic approximation, based on 2D electrons scattered by hard disks, randomly distributed, on the background of a smooth random potential.¹² In this more general case the magnetoresistance has also a parabolic *B*-dependence:

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} \simeq -\left(\omega_c / \omega_0\right)^2,\tag{5}$$

where $\omega_0 = 2^{1/4} \pi^{1/2} v_F (d^2 l_t l_L)^{-1/4}$, $l_L = v_F \tau_L$ is the transport mean free path due to the scattering by the smooth random potential, $l_{tr} = 1/(2Nd)$.

Recently, another numerical approach predicted for the 2D Lorentz gas with hard scatterers a nonanalytic negative magnetoresistance proportional to |B| at the classical level ($\beta \equiv \omega_c \tau_{tr} \ll 1$).¹³ The behavior of this anomalous MR was attributed to non-Markovian memory effects resulting from specific backscattering processes, ignored by the Boltzmann approach and that were not considered previously. The leading contribution to this negative and linear MR is attributed to low angle return events to a scatterer (1) after a single collision process with another scatterer (2). For $\omega_c \tau \ll 1$, the anomalous MR is approximated by the expression:

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} = -0.04(\omega_c/N). \tag{6}$$

The anomalous character of this MR was analyzed in Ref. 14 where a more complete theory was presented. According to this work small angle $\phi \leq \beta$ backscattering events, in single recollision processes, contribute dominantly with a quadratic *B* term to MR. This contribution is negative, and changes at very low fields $\beta \approx 0.05a/l$ to a linear *B* dependence (in this expression *a* is a measure of the effective radius of the scatterers and *l* is the transport mean free path). Equation (8) summarizes the asymptotic behavior of MR, for a 2D Lorentz gas formed by hard scatterers, in a wider range of magnetic fields, as presented in Ref. 14:

$$\frac{\Delta \rho_{xx}(B)}{\rho_{xx}(0)} = -\beta_0 f(z),\tag{7}$$

where $z = \beta / \beta_0$, $\beta_0 \equiv a/l$, and

$$f(z) = \begin{cases} 0.32z^2 \text{ for } z \leq 0.05, \\ 0.032(z - 0.04) \text{ for } 0.05 \leq z \leq 2, \\ 0.39 - 1.3z^{-1/2} \text{ for } z \to \infty. \end{cases}$$
(8)

A small linear negative decrease of magnetoresistance was observed previously in lattices with arrays of randomly distributed antidots,¹⁵ and for a long time, a reasonable theoretical explanation for this phenomenon has not arisen.



FIG. 1. (a) Scheme of the sample hall bar showing the location of the corrugated region. (b) Atomic-force microscope image of a small region (900 μ m²) of the corrugated region of the sample.

In this work we present the results of a study of the lowfield magnetoresistance in a two-dimensional electron gas constrained to move in a nonplanar topography originated by a random distribution of corrugations and defects that resulted from combined processes of prepatterning of antidot lattices and regrowth by molecular beam epitaxy (MBE). We report the observation of a large negative decrease of MR which is dominated by an anomalous linear character. From comparison with recent theory developments for the conductivity of a classical 2D Lorentz gas^{13,14} and numerical simulations performed by us, we argue that the observed LNMR is mainly due to non-Markovian memory effects that result from specific backscattering events.

II. EXPERIMENTAL RESULTS

A. Fabrication of samples

Samples for this work were fabricated by molecular beam epitaxy overgrowth of GaAs/Al_xGa_{1-x}As semiconductor high electron mobility transistor (HEMT) structures over semi-insulating GaAs substrates, oriented in the (100) direction. Samples were previously prepatterned with antidot lattices with 0.6 μ m period and 0.2 μ m average lithographic diameter. After regrowth, the samples with the patterned area were processed into Hall bars, with the nonplanar surface situated on one side, as shown in Fig. 1(a). The distance between voltage probes was 100 μ m and the width of the current channel is 50 μ m. We followed a preparation process similar to that described in Ref. 9. In a different way than samples presented in Refs. 9 and 16, the surface of the present samples consists of a random array of sharped corrugations with an admixture of "stripe-like" and "hill-like" topologies. The average amplitude of the corrugations is greater across the channel of the Hall bar than along it. Due to the fact that the two-dimensional electron sheet is close to the surface, carriers are constrained to move in this nonplanar topography.

Figure 1(b) shows an image of a square 900 μ m² region of the sample surface, obtained with an atomic-force microscope (AFM). It can be observed that the disordered stripes are mainly oriented along the length of the channel of the Hall bar. The average corrugation height of the ridges *h*, obtained from the AFM image, is about 650 Å.

B. Experimental magnetoresistance

A standard cryogenic superconducting magnetic system was used for magnetotransport experiments, they were performed at different temperatures near the temperature of the liquid helium. We employed the lock-in technique for detection, with an ac current not greater than 1×10^{-6} A. Measurements were performed with the magnetic field oriented perpendicular to the plane of the Hall bar in the positive and negative z directions around B=0 and also to the current I direction. The electron mobility in the planar region of the sample is $\mu=3.5 \times 10^5$ cm²/V s and in the corrugated region is about $\mu=2.1 \times 10^5$ cm²/V s. The electron concentration N_s is 5.5×10^{11} cm⁻².

Figure 2(a) shows the experimental relative decrease of $\Delta \rho_{xx} / \rho_{xx}(0) = [\rho_{xx}(B) - \rho_{xx}(0)] / \rho_{xx}(0)$ magnetoresistance around the zero magnetic field value taken at the temperature of 1.4 K. In a similar way as other two-dimensional systems with a certain degree of disorder, characterized by short range potentials, we observe a decrease of magnetoresistance, which is rather large and reaches up to 30% of its zero field value. At low field, in a very small region around $B=0(B \leq |0.02|T)$, when the dimensionless parameter $\beta \equiv \omega_c \tau_t \lesssim 0.4$, the longitudinal resistivity shows a parabolic behavior. Shortly after, the negative magnetoresistance turns to a completely linear dependence on the magnetic field between -0.2 and 0.02 T and 0.02 and 0.2 T. This linear behavior represents a decrease of up to 20% of $\rho_{xx}(B=0)$ $(0.4 \leq \beta \leq 4.2)$ which is a very remarkable fact since up to now a large phenomenon of this kind was not previously observed. After this interval, for $\beta > 4$, the magnetoresistance turns parabolic again but this time with a positive character and finally, in a short interval, remains as a background of the Shubnikov-de Haas oscillations.

Two-dimensional systems grown on prepatterned GaAs substrates have been recently used to study the electron dynamics in nonhomogeneous magnetic fields.^{9,16} In these works, the regrowth processes resulted in regular stripes with a well defined periodicity along or across the Hall device channel (contrary to the present samples characterized by a shorter lattice period), and magnetotransport experiments were mainly focused on the study of the electron dynamics when the magnetic field is oriented parallel or tilted in relation to the sample substrate. In the samples of the present work, the magnetic field is oriented strictly perpendicular to the sample surface and also to the current direction. If we



FIG. 2. (a) Experimental relative decrease of magnetoresistance taken at 1.4 K, for the sample with random corrugations. (b) The negative magnetoresistance shows a parabolic behavior in a very small region around B=0, where the classical parameter $\beta \leq 0.4$. (c) and (d) A large linear *B* dependence is observed between $|0.4| \leq \beta \leq |4.2|$ for negative and positive field values around B=0.

consider a corrugated surface defined by f(x, y, z)=z-f(x, y)=0, with a magnetic field $B_{ext}=(0, 0, B)$ oriented perpendicularly to the substrate, the electron motion is only sensitive to the normal component of the field B_{eff} to the sample surface. This normal component of the magnetic field is given by the projection of the external uniform magnetic field vector over the gradient vector $\vec{\nabla}f(x, y, z)$, this can be expressed as

$$B_{eff}(x, y, z) = \frac{\vec{B} \cdot \vec{\nabla} f}{|\vec{\nabla} f|},$$
(9)

where $\vec{\nabla} f = (df/dx, df/dy, 1)$. Expression (9) can be rewritten more explicitly as

$$B_{eff} = \frac{B}{\sqrt{1 + \left(\frac{df}{dx}\right)^2 + \left(\frac{df}{dy}\right)^2}}.$$
 (10)

According to this expression B_{eff} fluctuates in amplitude along and across the sample surface without changing sign. From the AFM data we obtain that the average distance between adjacent hills or stripes d is about 50 000 Å which is larger than the average height of the corrugations (h=650 Å). According to this, df/dx and df/dy can be approximated by $h/d \ll 1$, from this we assume that the amplitude of the magnetic fluctuation is negligible and does not contribute significatively to the scattering phenomena in our samples.

Thus carrier scattering in our samples should be mainly due to strong scatterers such as corrugations and defects (remaining antidots) of a short-range nature, and a long-range random potential due to ionized impurities. It was shown that the interplay of these two kinds of potentials conduces to a nonsaturating negative magnetoresistance in a Lorentz gas composed of hard-core reflecting disks.³ However, this model does not account for the large linear anomalous character of the NMR observed in our experiments.

We used the recent theoretical results obtained by V. Cheianov *et al.*¹⁴ to analyze our experimental results. Figure 2(b) shows the very low magnetic field region of the negative magnetoresistance shown in part (a). The NMR shows an analytical behavior when $\beta \rightarrow 0$ ($B \leq |0.02|T$), and this part of the curve was fitted according to the first part of the expression (8), we extract the parameter a/l=5.3, where *l* is of the order of 2.5 μ m and the effective radius of the scatterer yields $a=13.2 \ \mu$ m.

Figures 2(c) and 2(d) show a linear fit for the NMR between $|0.02|T \le B \le |0.2|T$, this corresponds to the region between $0.42 \le \beta \le 4.2$, according to these fittings we obtain $\Delta \rho_{xx} / \rho_{xx}(0) = [\rho_{xx}(B) - \rho_{xx}(0)] / \rho_{xx}(0) \approx -0.98B$. By comparison with the second part of expression (8), and keeping the factor 0.032, we obtain the value $l=3.7 \ \mu m$. From the fitting of the parabolic region we obtain the average value of the a/l ratio which is greater than one due to the greater cross section of the corrugations in relation to the mean free path. From comparison with Eq. (8) we find qualitative agreement between our experimental results and the theoretical model, which indicates that the main contribution to the observed NMR is due to non-Markovian effects resulting from low angle 1-2-1 single recollision processes in backscattering of electrons by corrugations. However, a certain difference between the theoretical model and our experimental results is related to the order of magnitude of the full variation of $\Delta \rho_{xx} / \rho_{xx}(0) = [\rho_{xx}(B) - \rho_{xx}(0)] / \rho_{xx}(0)$. In the case of the Lorentz gas composed of hard scatterers, with disk radius a this variation is of the order of β_0 . In the case of our measurements the whole decrease of MR reaches up to 30% the value of $\rho_{xx}(B=0)$. This phenomenon that may be closely related to a specific "corridor effect" for corrugations, that should present greater cross sections for electron scattering, demands further calculations for the classical electron dynamics in these particular 2D disordered systems.

Linear negative magnetoresistance, at very low magnetic field, was first reported by Gusev *et al.*¹⁵ in measurements in microscopic and mesoscopic antidot lattices with random distribution of scatterers. In that work the anomalous magnetoresistance showed a temperature independent character and its relative linear decrease reached a maximum value of about 3%. At the time, a consistent theoretical model was not available for a proper explanation of this phenomena. In the samples of the present work we observed a temperature dependence of the low field magnetoresis-



FIG. 3. Negative magnetoresistance at low magnetic field, for different temperatures between 1.4 and 31 K.

tance, this phenomenon manifests itself as a gradual increase of the parabolic behavior, over the linear character, as |B| grows. Figure 3 shows the magnetoresistance curves, at low field around B=0, for different temperatures between 1.4 and 31 K. We observed that the negative parabolic behavior around B=0 increases rapidly for temperatures above 8.5 K, this means that the initial region $B \leq |0.02|T$ for T=1.5 K extends up to $B \leq |0.06|T$ for T=23 K and $B \leq |0.1|T$ for T=31 K. Due to a simultaneous increase of the saturation value with the temperature, the linear region of the negative magnetoresistance shrinks but the linear trend remains in a small region at temperatures above 30 K corresponding to a decrease of up to 10% of the zero field resistivity.

III. NUMERICAL CALCULATIONS

For a proper description of the low field magnetoresistance in our samples it is necessary to simulate numerically the dynamics of two-dimensional electrons constrained to move in a randomly shaped nonplanar topography in the presence of a uniform perpendicular magnetic field.¹⁷ This task is under development and is not presented in this work. Instead, as an alternative approach, we assume that the electron motion, influenced by the uniform magnetic field, in the nonplanar topology formed by the random distribution of corrugations and defects, can be compared with the simulation of a two-dimensional Lorentz gas composed of electrons, under the influence of a perpendicular uniform magnetic field. The array of nonoverlapping scatterers is characterized by Gaussian potentials, distributed randomly, as shown schematically in Fig. 4. As we are interested in the classical nature of transport in this system, we used linear response theory for the calculation of magnetoresistance ρ_{xx} through a different numerical approach.

In the next we consider the classical approximation for the dynamics of an electron in an array of two-dimensional random potentials and, in the presence of magnetic field $\vec{B} = (0,0,B_z)$, perpendicular to the 2D plane. The Hamiltonian of a single electron confined to the x-y plane by a



FIG. 4. Scheme of an array of two-dimensional antidots, formed by electrostatic Gaussian potentials distributed randomly. E_F represents the Fermi energy and \vec{B} the magnetic field vector.

potential $U_w(x, y)$, and an array of random scatterers $U_{2D}(x, y)$, is found to be

$$H = \frac{1}{2m^*} (\vec{p} - e\vec{A})^2 + U_w(x, y) + U_{2D}(x, y), \qquad (11)$$

where the potential vector is written as $\vec{A} = (-By/2, Bx/2, 0)$, and the random antidot potential of the antidots U_{2D} is simulated by the expression

$$U_{AD}(x,y) = \sum_{i=1}^{M} U_0 \exp\left[-\left(\frac{x}{\Gamma_x}\right)^{\gamma}\right] \cdot \exp\left[-\left(\frac{y}{\Gamma_y}\right)^{\gamma}\right],$$
(12)

where *M* is the number of antidots, U_0 is the maximum amplitude of the potential of each scatterer, the parameters Γ_x and Γ_y account for the antidot diameter at the Fermi energy, and γ allows us to vary between soft and hard potential profiles, for our calculations we employed $\gamma \sim 2-6$. Due to computational limitations we introduce periodic conditions by enclosing the Lorentz gas into a square box confined by hard potential walls of the type:

$$U_w = U_x x^n + U_y y^m, \tag{13}$$

where U_x and U_y are the maximum amplitudes of the walls at the borders and *m* and *n* are integers that account for the steepness of the confining potential walls. We used dimensionless variables in a similar way as those explained in Ref. 18. Four equations of motion are obtained and they were numerically integrated to obtain the electron trajectories. According to classical linear response theory the conductivity of the 2D electron gas is given by

$$\sigma_{ij} = \frac{N_s e^2}{E_F} \int_0^\infty \langle v_i(t) v_j(t=0) \rangle_{\Gamma} e^{-t/\tau} dt, \qquad (14)$$

where N_s is the electron concentration, E_F is the Fermi energy, $\langle v_i(t)v_j(0)\rangle_{\Gamma}$ is the velocity-velocity correlation function double averaged over phase space Γ , and the indices *i* and *j* stand for the *x* and *y* direction, respectively. The presence of additional scattering by phonons or defects is included through the electron mean scattering time τ , where



FIG. 5. Relative magnetoresistance for a two-dimensional Lorentz gas obtained by numerical integration.

the probability of an electron not suffering a collision within the time interval [0,t] is given by $e^{-t/\tau}$.

From the numerical computation of the conductivity tensors σ_{xx} and σ_{xy} , as a function of the perpendicular magnetic field, we are able to determine the longitudinal ρ_{xx} and transverse ρ_{xy} resistivities through the expressions

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}\sigma_{yx}},$$
(15)

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}\sigma_{yx}}.$$
 (16)

In order to calculate conductivity, we generate an ensemble of electron trajectories, inside a square region of side a_l , where a_l is the antidot lattice period, chosen to be equal to unity. This region contains 400 scatterers, with Gaussian potentials, placed according to a uniform distribution. The ratio a/l (where a is the radius of the scatterer at the Fermi energy) was chosen to be 0.001, and the mean free path $l=(N2a)^{-1}$ was $6.3a_l$. *N* is the two-dimensional concentration of scatterers. For the integration process we generate 200 trajectories for each particular configuration of scatterers, after this a new configuration is established and the process is repeated up to 10 times. Due to computational limitations the very low field region of magnetoresistance $\beta \leq 1$, where $\beta \equiv \omega_c \tau_{tr}$, is not accessible in our simulations.

Figure 5 shows the numerical results for the relative magnetoresistance obtained through our simulation. We observed a large linear negative decrease of MR between $1.0 \le \beta \le 6$, for $\beta \ge 6$ the resistivity turns parabolic and positive reaching a saturation for $\beta \ge 10$. Greater values for the ratio a/l were not possible to obtain in our calculations due to the rapid increase of the smoothness of the Gaussian potentials for greater values of the parameters Γ_x and Γ_y . In conclusion these numerical calculations showed that the linear response approach for the conductivity detected correlations that may be attributed to memory effects from backscattering events due to the large linear decrease of magnetoresistance for $1.0 \le \beta \le 6$.

IV. SUMMARY

In summary, we have studied the low field magnetoresistance of a two-dimensional system constrained to move in a nonplanar topography composed of random corrugations and defects, contrary to the prediction of the Boltzmann-Drude approach the MR of this disordered system shows a dominant large linear negative decrease. The comparison of our experimental curves with a recent theoretical model for the MR of a 2D Lorentz gas with a random array of scatterers indicates that the main corrections to the conductivity in our samples are due to non-Markovian memory effects resulting from specific recollision processes.

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